Global Stage-1 Optimization for Stellarator Design

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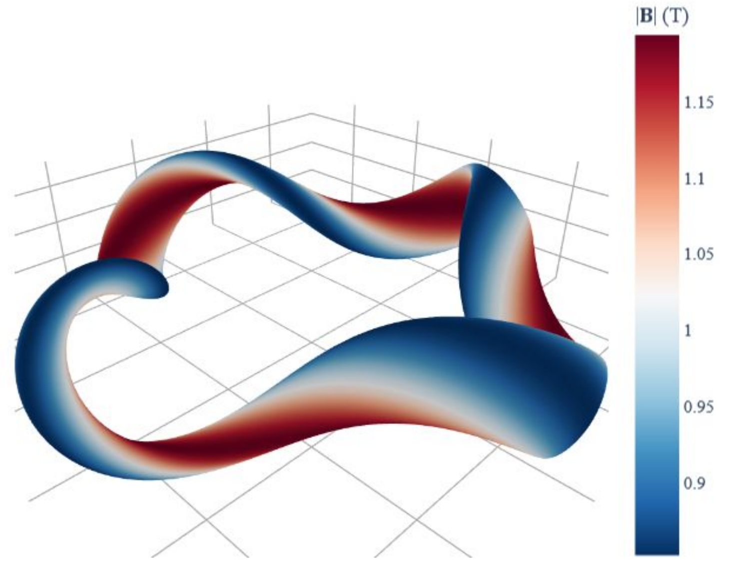
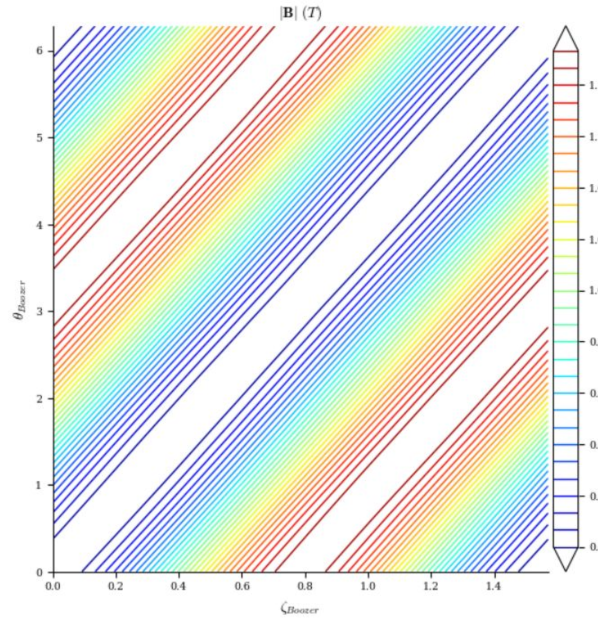
Introduction

Stellarators

- An asymmetric analogue of the tokamak, stellarators are plasma containment devices which are strong candidates for sustained nuclear fusion.
- Asymmetry improves containment and stability without the need for externally driven plasma currents.

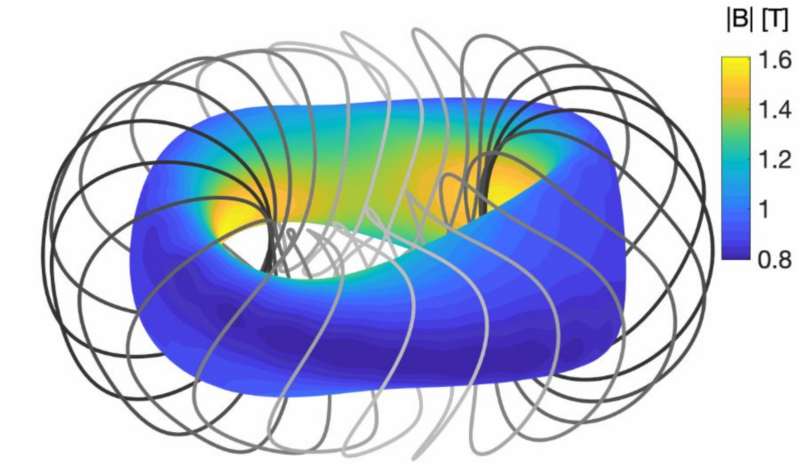




Hidden Symmetries

- One desirable property for stellarators with strong containment is "quasisymmetry," a special symmetry in the magnetic field magnitude along the field lines.
- Stellarators can be toroidally (QA), poloidally (QP), or helically (QH) quasisymmetric.

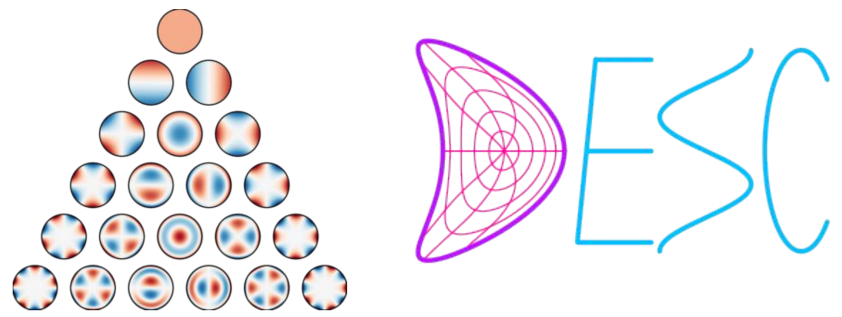
Stage-1 Optimization



Matt Landreman - UMD Stellarator Group

Finding Optimal Stellarators

- To find desirable stellarators, we perform optimization over the space of plasma boundaries. This is called "stage-1 optimization."
- Then, stage-2 optimization finds coils which produce the optimized plasma boundary.



Performing Stage-1

- Apply local or global optimization algorithms to optimize for a weighted combination of physical objectives.
- At each stage-1 step:
  - Start with a plasma boundary
  - Compute the equilibrium inside using an equilibrium solver like VMEC or DESC
  - Compute physical objectives such as quasisymmetry, aspect ratio, volume, rotational transform, …

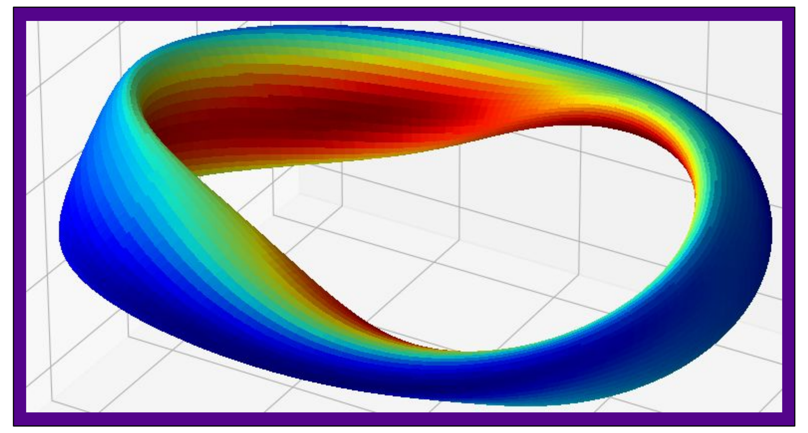
$$F(\vec{x}) = w_{QS} f_{QS}(\vec{x})^2 + w_{AR}(f_{AR}(\vec{x}) - \bar{f}_{QS})^2 + w_\iota(f_\iota(\vec{x}) - \bar{f}_\iota)^2 + \cdots$$

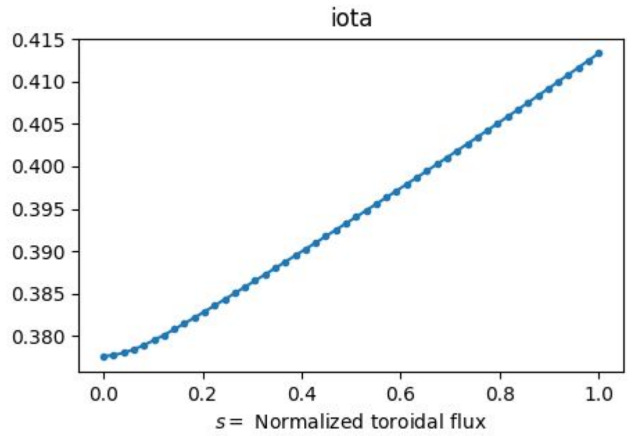
$$f_{QS} = (M\iota - N)(\mathbf{B} \times \nabla\psi) \cdot \nabla B - (MG + NI)\mathbf{B} \cdot \nabla B$$

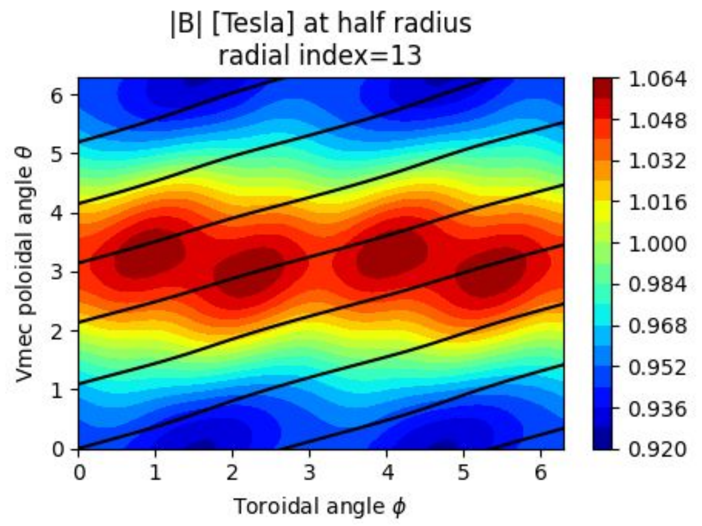
Local Stage-1 Optimization

Local Optimization

- The standard method for stellarator optimization is local optimization: after each step, perturb the boundary in a direction chosen by a gradient-descent like algorithm (e.g. BFGS).
- We parameterize a boundary by its Fourier modes. Local algorithms do a least-squares search on the objectives in a high dimensional space of valid plasma boundaries which admit MHD equilibria with nested flux surfaces.







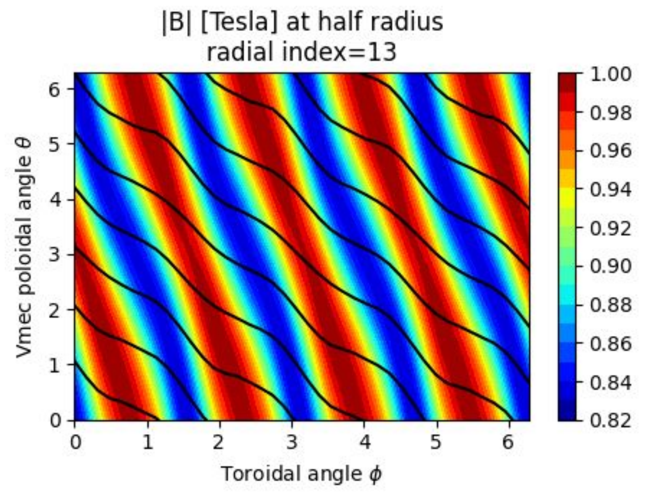
Problems and Solutions

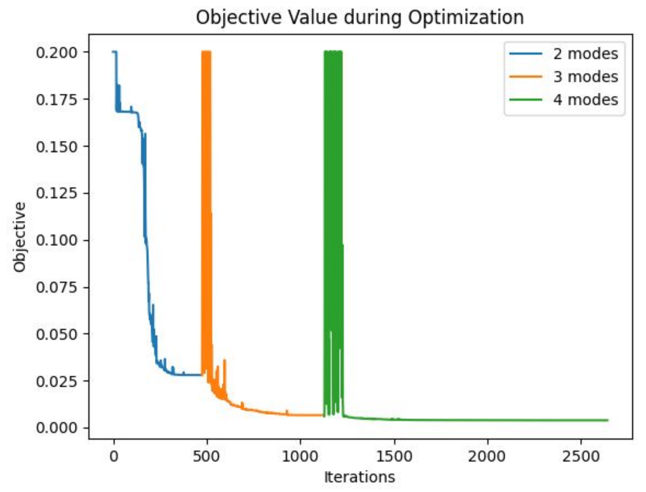
- If we start with a high number of modes, the local search finds poor local minima.
  - Use a "continuation method": first optimize with few modes, then increase the number of modes and optimize again, and so on.
- Local searches perform poorly on rough objectives with many local minima (i.e. nonlinear turbulent transport).
  - Many directions: smoother proxy objectives, stochastic optimization, **global optimization**.

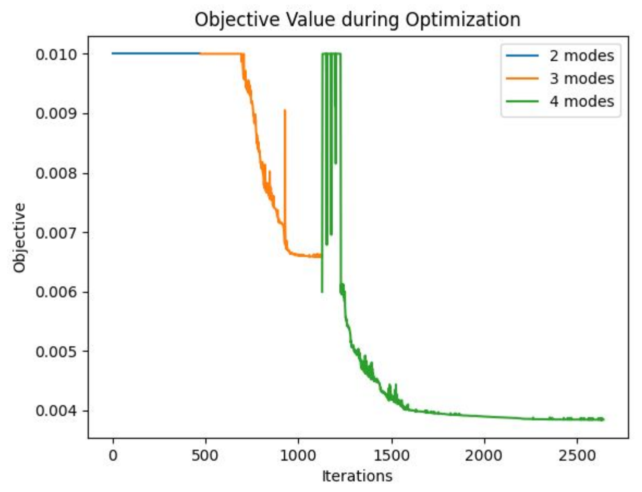
Global Stage-1 Optimization

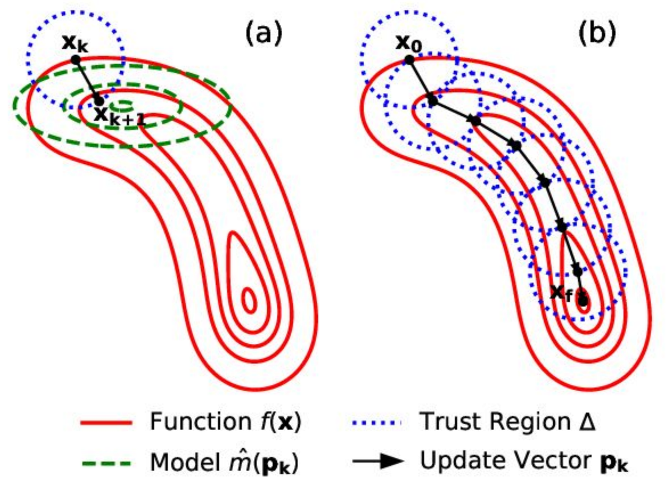
Global Optimization

- We apply a global optimization algorithm, the Powell's Derivative-Free Optimization (PDFO) solver COBYLA, to stage-1 optimization for vacuum stellarators in both Simsopt with the VMEC equilibrium solver and DESC.
- PDFO performs optimization by constructing linear models for the nonlinear objective on dynamic trust-regions.
  - Capable of handling nonconvex and even rough objectives
  - Can explore more of the optimization space (e.g. optimizing over higher modes from the start)

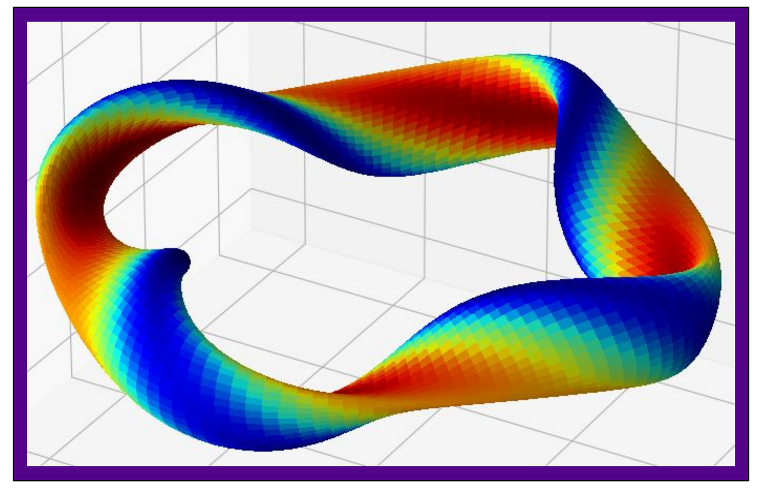








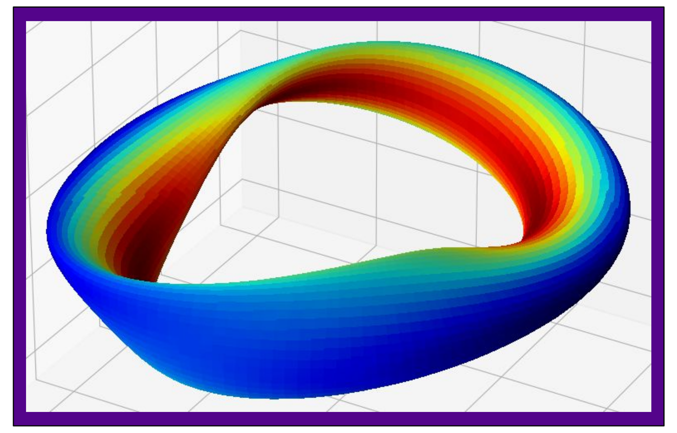
Robert Smith, Lucas Hofer, Milan Krstajić: JAXFit

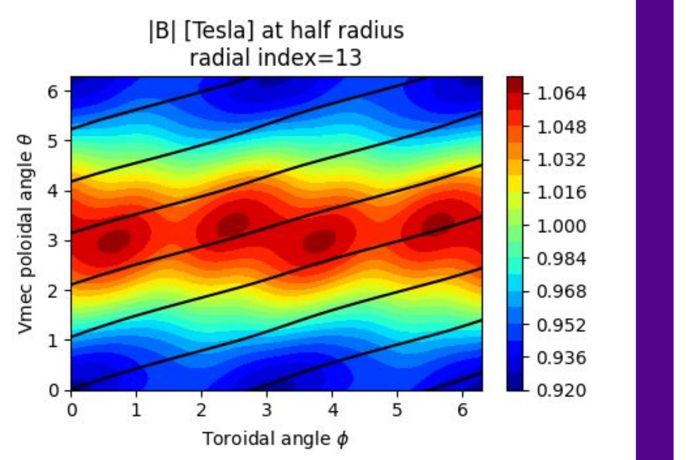


Global Optimization Results

VMEC, Low Modes

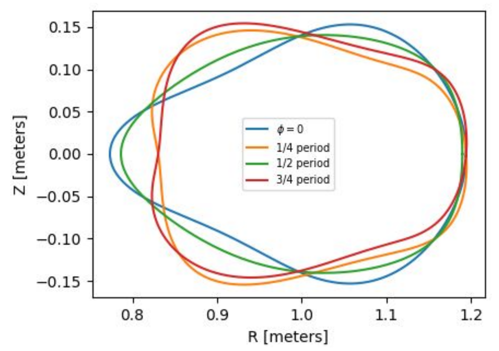
- Similar stellarators to those obtained by local optimization.
- Convergence takes ~5x as many objective evals.

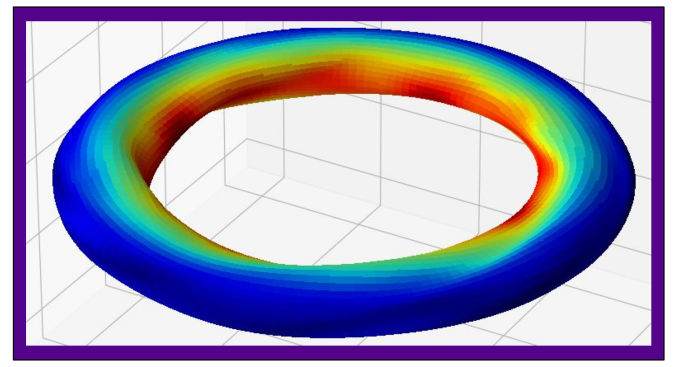


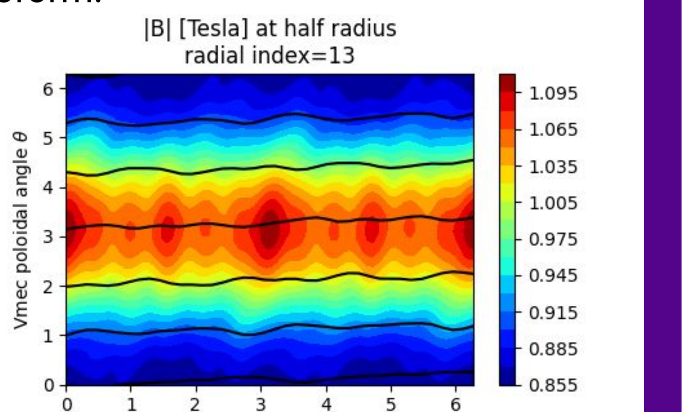


VMEC, High Modes

- New perturbed tokamak-like minima with nontrivial rotational transform.

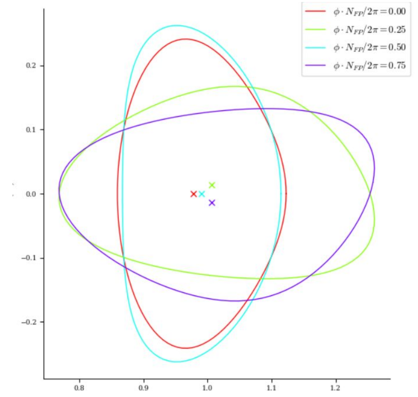


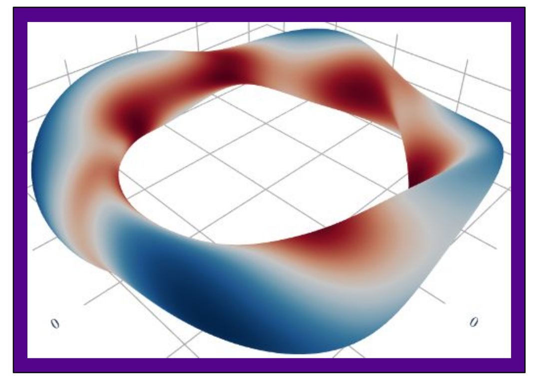


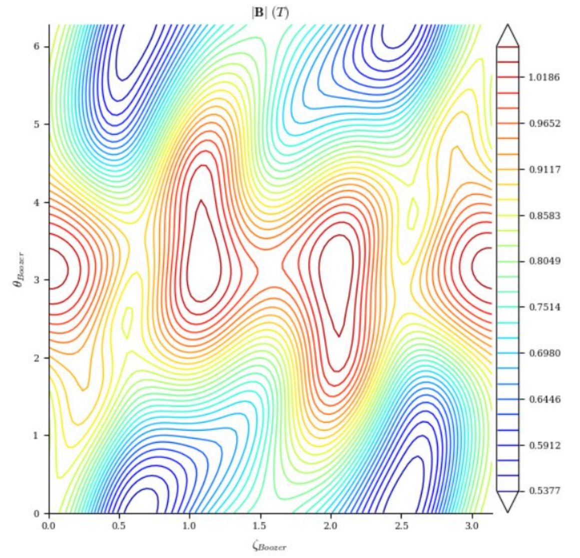


DESC, High Modes

- Fast convergence, but still far from quasisymmetry! Requires further dynamic tuning.







Barriers and Further Work

Barriers: VMEC

- When solving with VMEC, PDFO converges but performs slowly compared to local BFGS optimizers.
  - Because PDFO explores a large subset of the optimization space, many plasma boundaries it attempts to resolve fail to contain equilibria with nested flux surfaces. The large quantity of failed VMEC calls slows convergence significantly as compared to local optimizers which quickly escape poor regions of the optimization space.
  - Solution: the DESC equilibrium solver uses a pseudospectral basis instead of finite differences, resolving failed equilibria just as quickly as nested flux surfaces.

Barriers: DESC

- When solving with DESC, PDFO performs quickly but converges to suboptimal local minima.
  - For example, initializing with a tokamak and optimizing for nontrivial rotational transform and quasisymmetry, PDFO explores the optimization space but ultimately returns to the tokamak.
  - Proposed solution: tune the weights and trust radius dynamically over multiple optimization calls.

Future Work

- Tune the DESC optimizer to converge to equilibria which match the quality of local solvers.
- Explore the optimization space more carefully to locate new and interesting stellarators.
- Implement new global optimizers in DESC: TuRBO, dTuRBO (we should use derivative information!).
- Try optimizing for rougher objectives which capture nonlinear phenomena

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