Stochastic Ideas for Stellarator Optimization

Mason Haberle
Courant Institute
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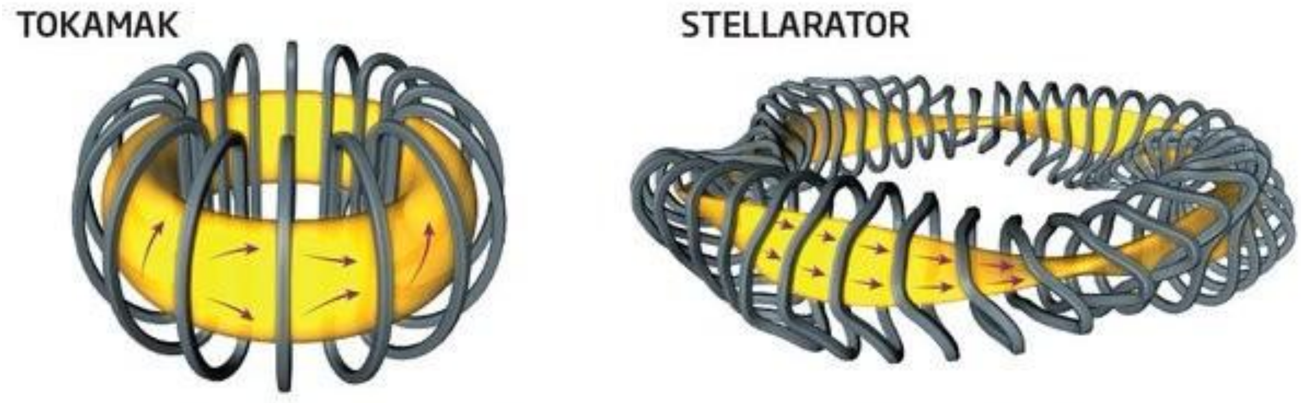
The Goal: Fusion

- Plasma containment
  - Tokamaks: axisymmetric
  - Stellarators: asymmetric
- Fusion: Deuterium + Tritium □ Helium + Neutron
- Need a super-hot, super-fast, super-controlled reaction.



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What is a plasma?

- Ionized gas: ions and electrons
- Quasineutrality
  - Electric field shielding
  - Still induced magnetic fields
  - Still induced electric fields
- Types
  - Astrophysical
  - Fusion
  - Low-temperature

KINETIC MODEL

Vlasov Equation

↓

Gyrokinetics

↓

Two-fluid Model

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Magnetohydrodynamics

↓

MHD Equilibrium

FLUID MODEL

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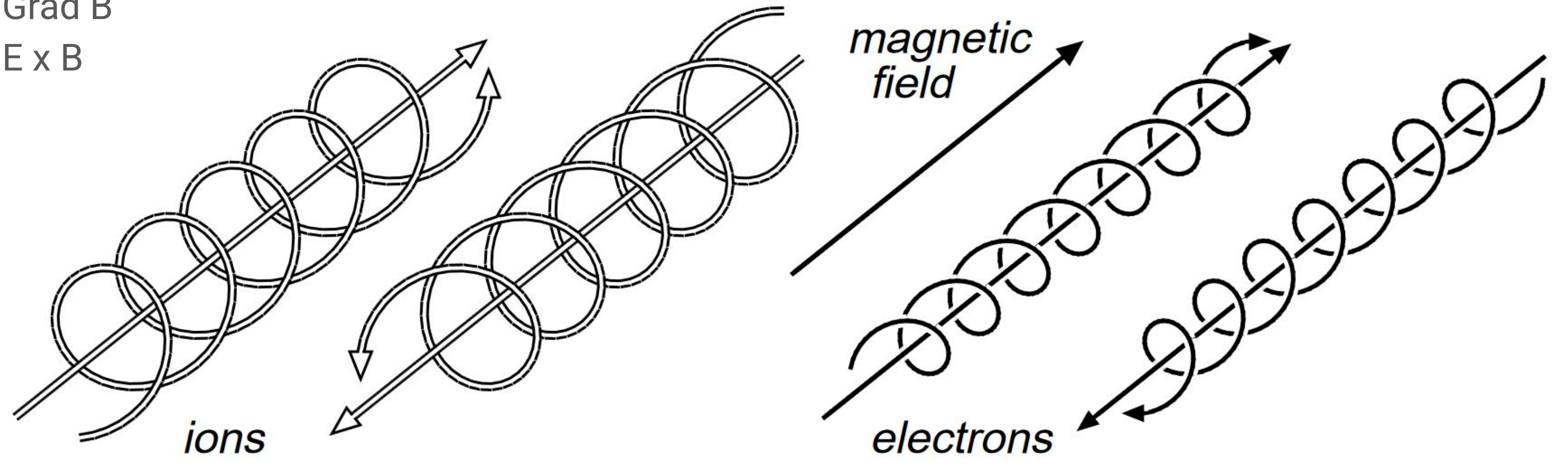
Dynamics

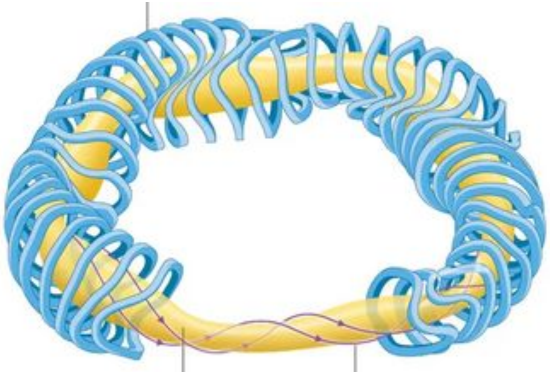
- Helical trajectories
- Follow B-field lines
- Drifts
  - Curvature
  - Grad B
  - E x B

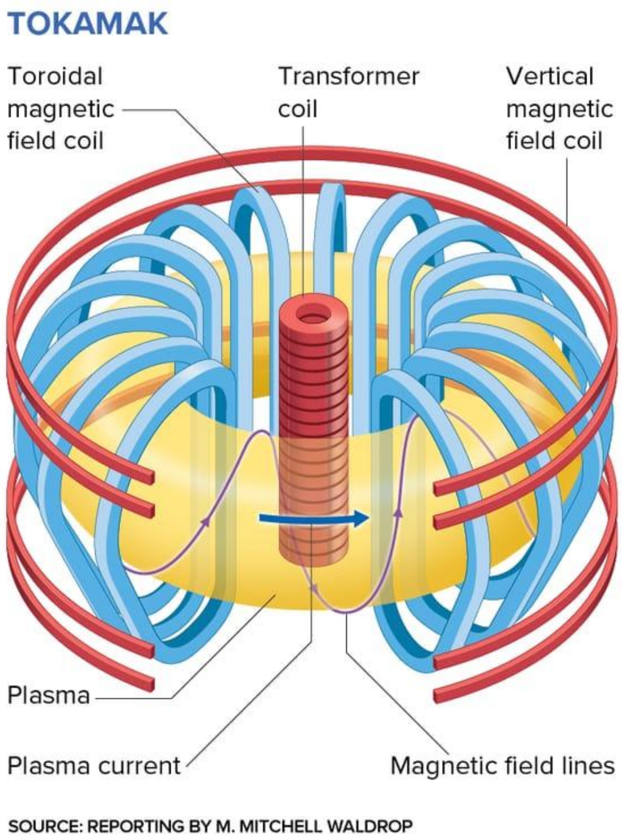
$$\dot{R}_\perp = C_\kappa(B \times \kappa) + C_{\boldsymbol{\nabla}}(B \times \boldsymbol{\nabla}|B|) + C_E(E \times B)$$



Toroidal Confinement

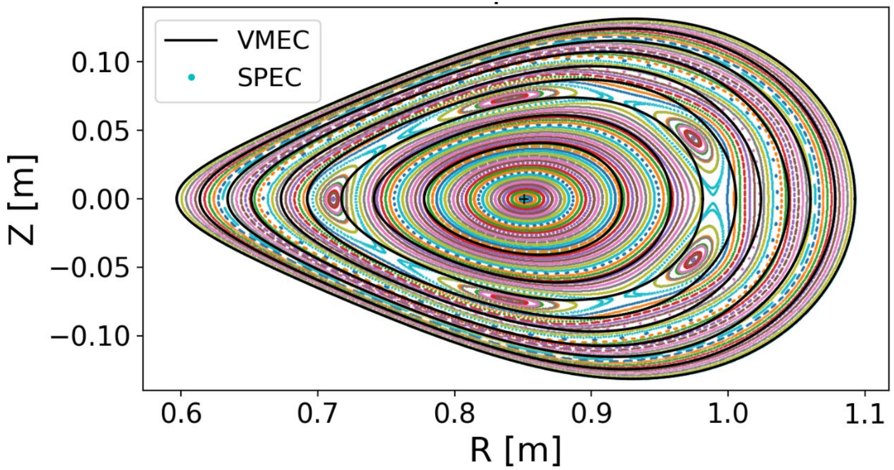
- Wrap field lines into circles.
  - Problem: drifts
  - Solution: helical B-field “rotational transform”
- Tokamaks
  - Drive a toroidal current using induction
- Stellarators
  - Asymmetry to produce rotational transform

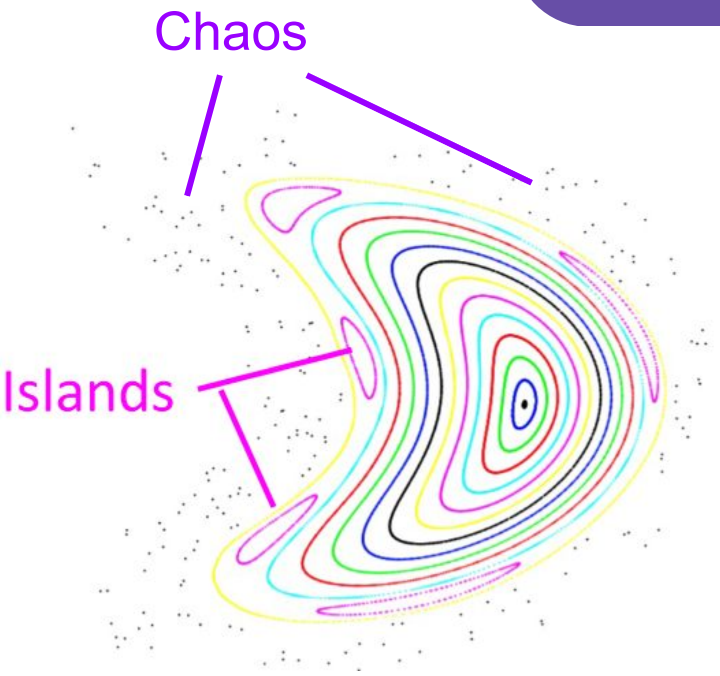




Stellarator Orbits

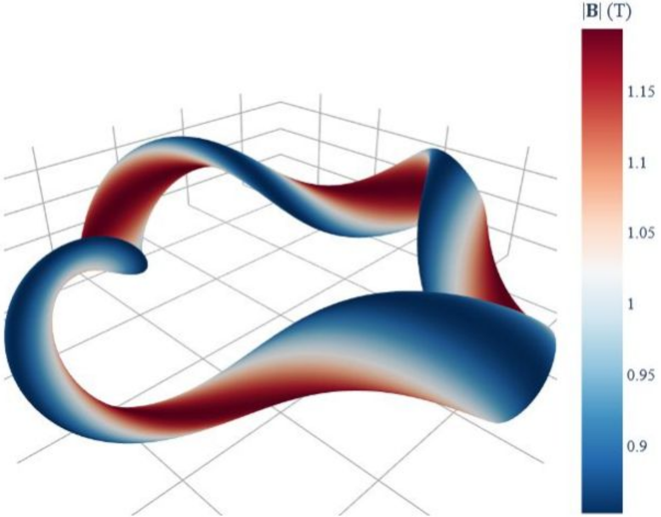
- Goal: nested flux surfaces
- Magnetic islands
- Chaotic field lines

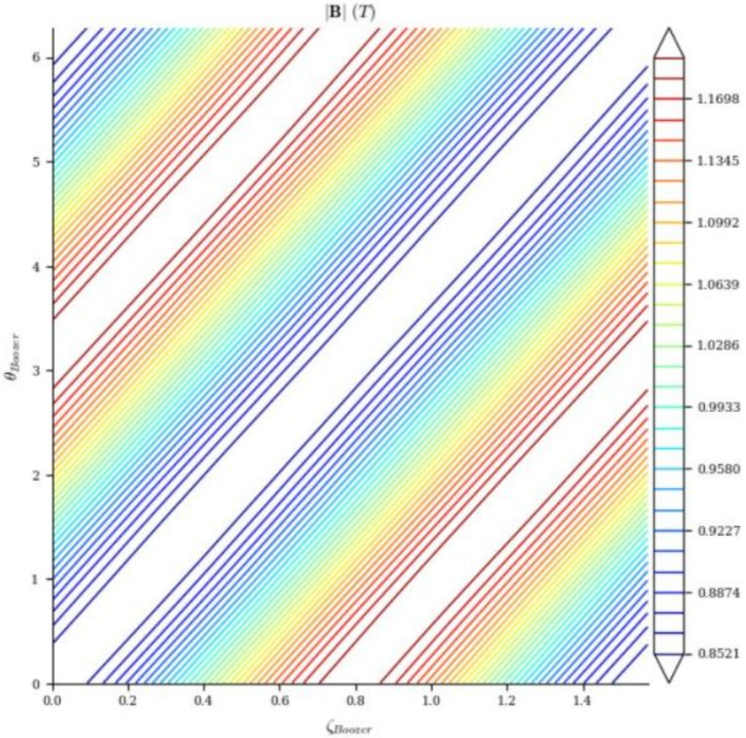




Quasisymmetry

- “Hidden symmetry” in the B-field
- Noether’s theorem: perfect containment
- All the drifts cancel over toroidal orbits.





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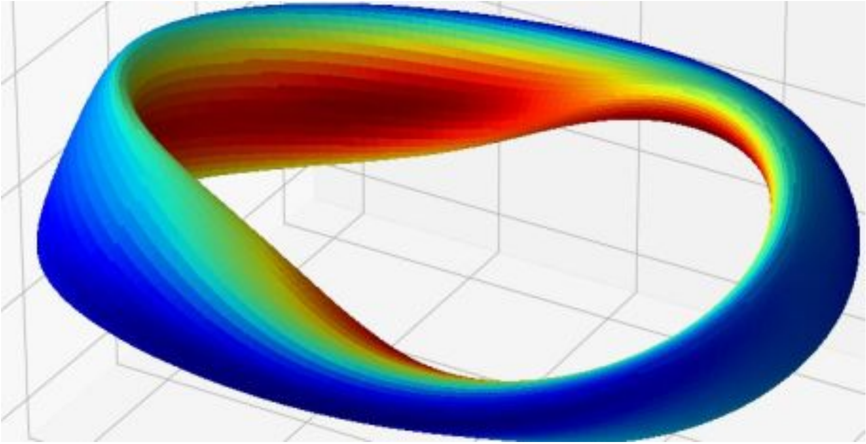
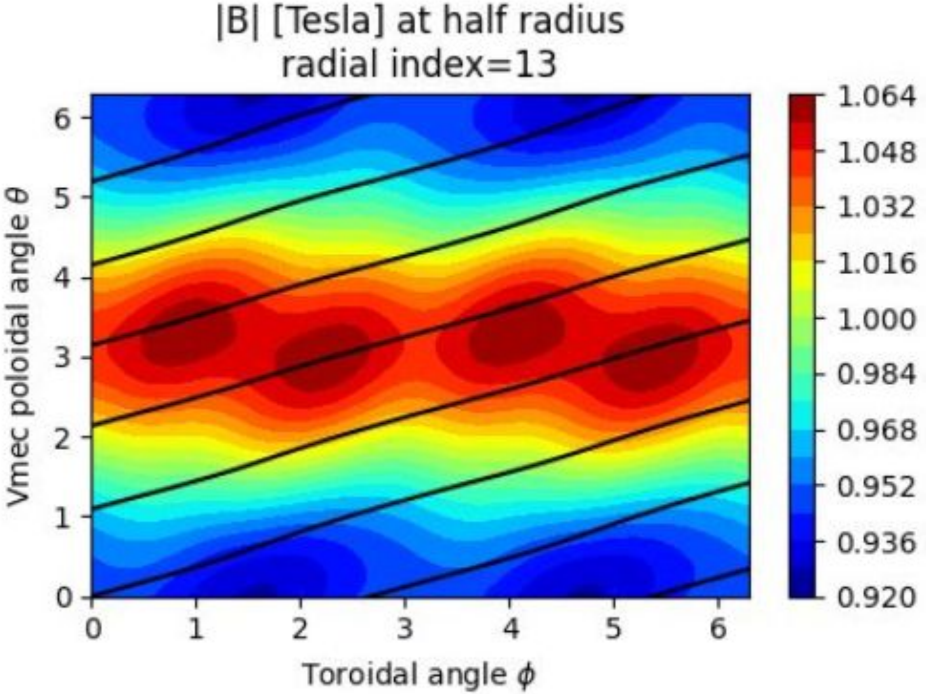
Quasisymmetry





Stellarator Optimization - Stage 1

- Parameterize plasma boundary by Fourier modes
- Pick physical objectives (quasisymmetry, rotational transform, volume…)
- Steps to optimize:
  - Start with a plasma boundary
  - Solve the MHD equilibrium inside (weak formulation)
  - Compute objectives
  - Perturb to a new plasma boundary
- Usually use local optimization (BFGS)

☐ **Result: A good plasma boundary**

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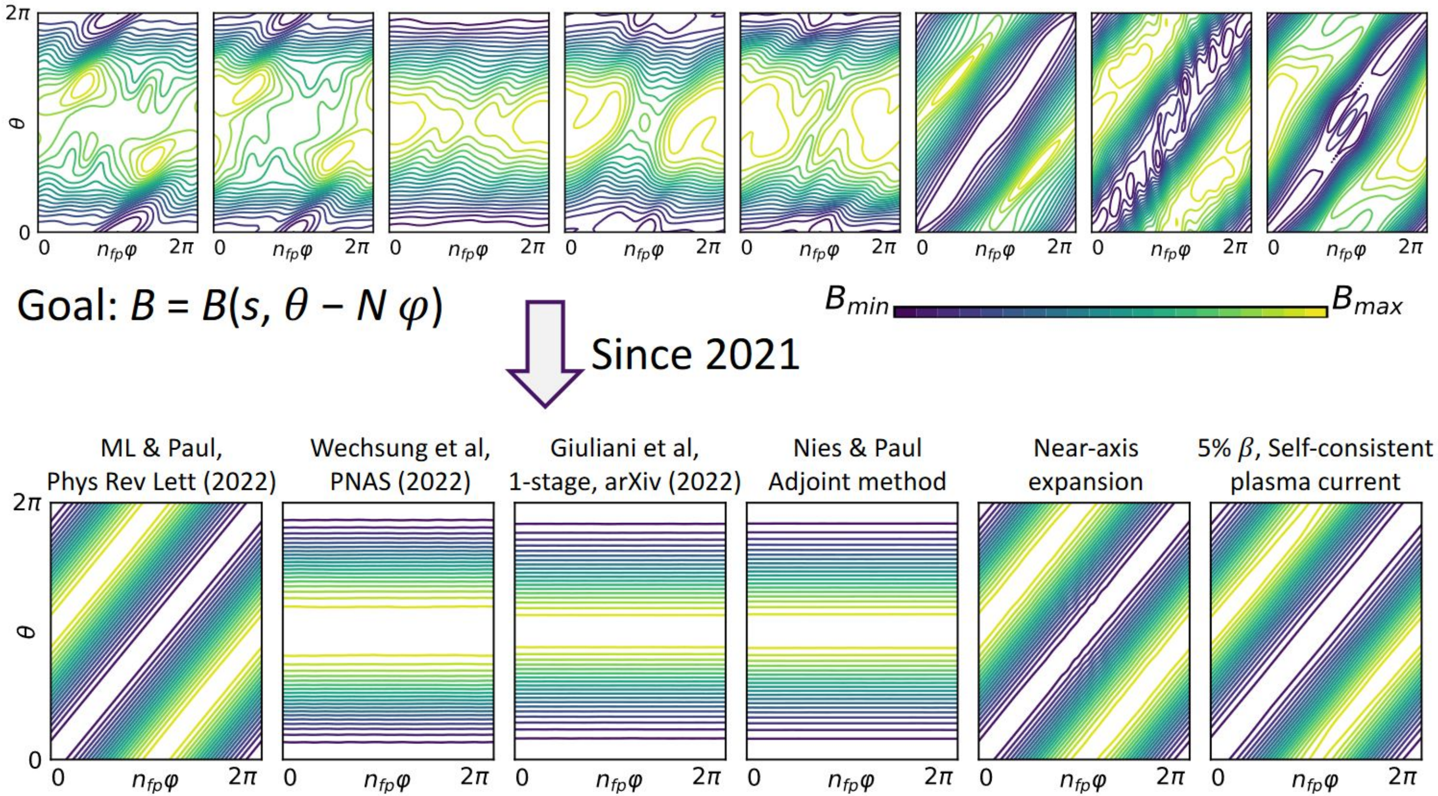

(slide shamelessly stolen from Matt Landreman)

Stellarator Optimization - Stage 2

- Parameterize coils by Fourier modes
- Pick coil constraints (length, separation, curvature…)
- Steps to optimize:
  - Start with coils
  - Solve for the B-field (Biot-Savart)
  - How parallel is B with the plasma boundary?
  - Perturb the coils
- Still local optimization

□ Result: Coils to shape your plasma

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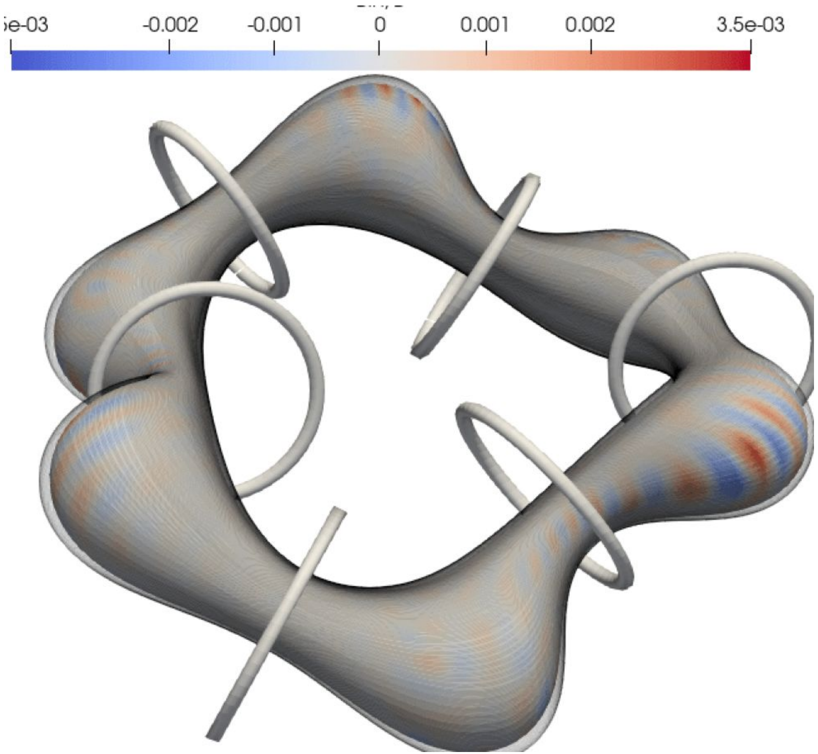
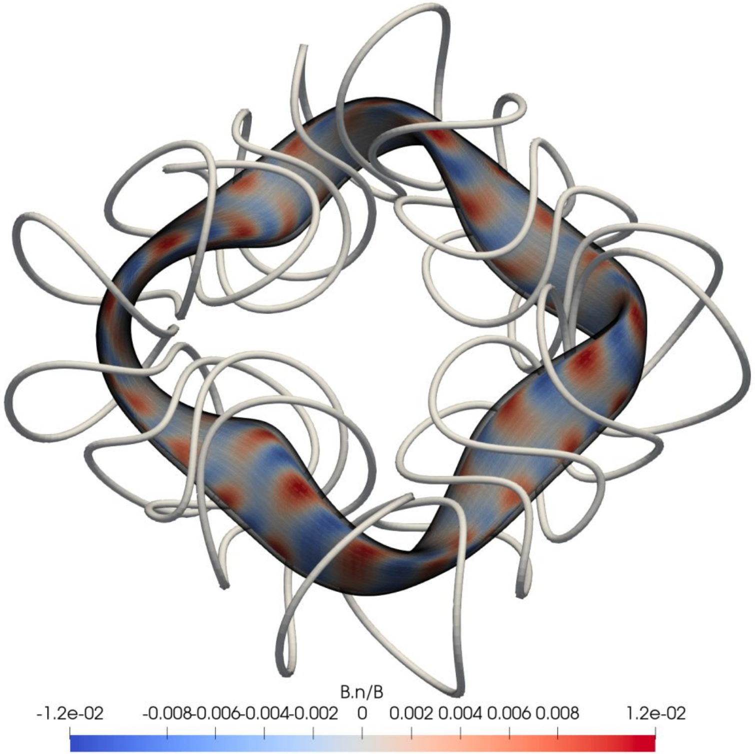

-0.002
-0.001
0
0.001
0.002
3.5e-03


B.n/B
-1.2e-02
-0.008-0.006-0.004-0.002
0
0.002 0.004 0.006 0.008
1.2e-02

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Robustness

- Stage-2 perturbations - not a big deal
  - Biot-Savart integral is a compact operator
  - Small errors to plasma boundary
- Stage-1 perturbations - really bad
  - Quasisymmetry and other confinement objectives are very sensitive to perturbations in the plasma boundary
  - Nonlinear effects: chaos, turbulence onset

$$f_{\mathrm{QS}} = (M\iota - N)(\mathbf{B} \times \nabla\psi) \cdot \nabla B - (MG + NI)\mathbf{B} \cdot \nabla B$$

Small Coil Errors

⎕ Small Plasma Errors

⎕ Big Loss in Confinement!

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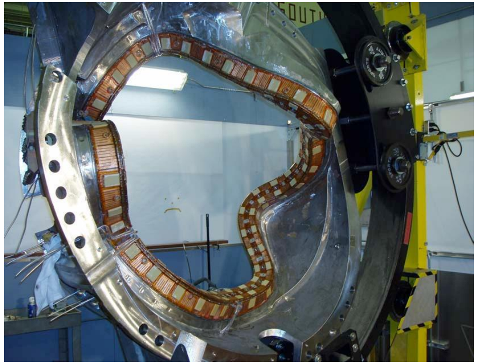
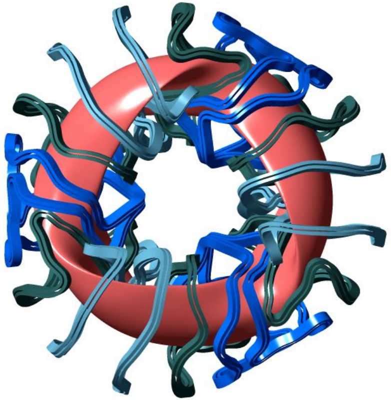
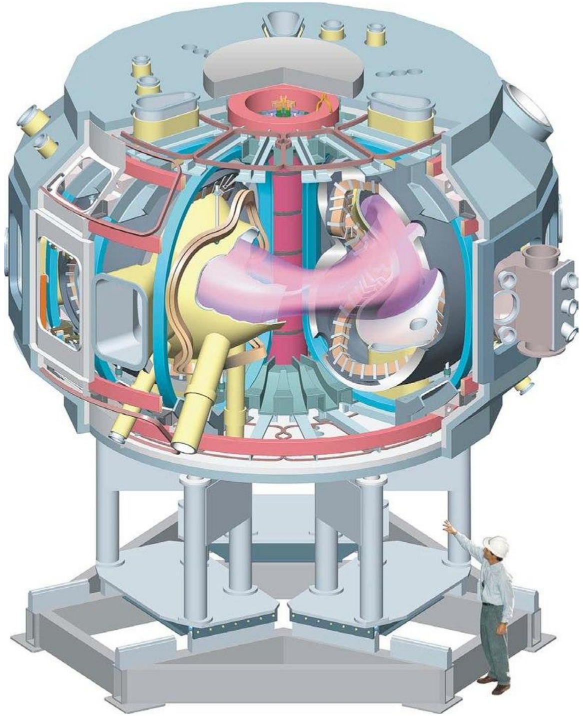
This Matters

- NCSX - Princeton's stellarator: 18 complicated coils
- 1.5 mm tolerance, tons of fancy instruments used
- $100m budget, eventually $170m estimate







CANCELLED.

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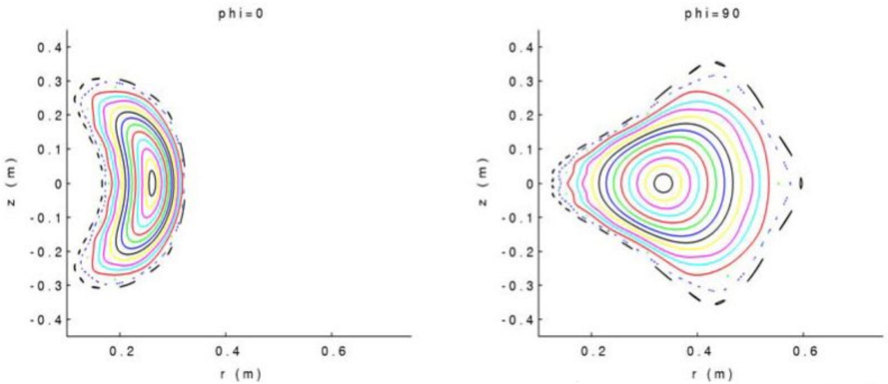
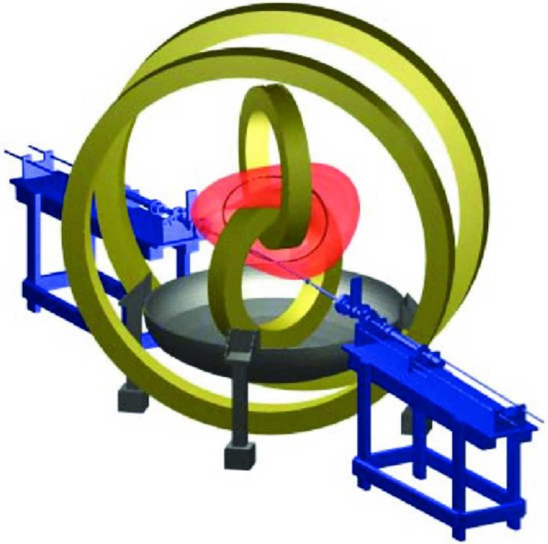
Stochastic Stage-2

- Solving instabilities
  - Find more robust coils
  - Explore more of the optimization space
- 3 pronged approach
  - Stochastic coils
  - Global stochastic search
  - Local stochastic refinement



The CNT.



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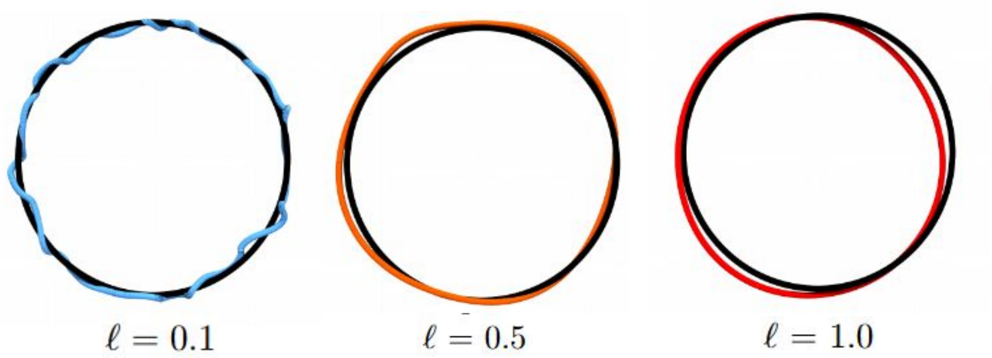
Stochastic Coils

- Add random, spatially correlated perturbations to the coils.
  - Scaled gaussian perturbations of Fourier modes
  - Random perturbations of coil angles and positions
- Evaluate expectation of objectives on the perturbed coils.
  - Stochastic objective: flux surface alignment
  - Deterministic objective: coil regularity
- Keep track of variance.

$$\boldsymbol{X}_P(\mathbf{x}, t) := \boldsymbol{X}(\mathbf{x}, t) + \boldsymbol{g}(t) \sim \mathcal{GP}(\boldsymbol{X}(\mathbf{x}, t), \kappa_{\text{per}}(t, t'))$$



$$\kappa_{\text{per}}(t, t') = h \exp\left(-\frac{2\sin^2\left(\frac{|t-t'|}{2}\right)}{\ell^2}\right)$$

$$f(\mathbf{x}) = \mathbb{E}[\omega_B f_B(\mathbf{x} + \boldsymbol{U})] + \omega_L f_L(\mathbf{x})$$

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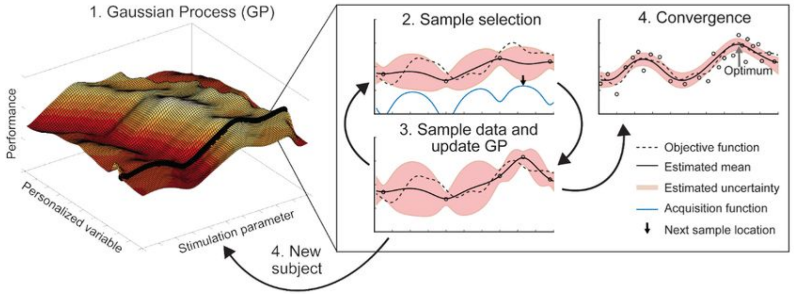
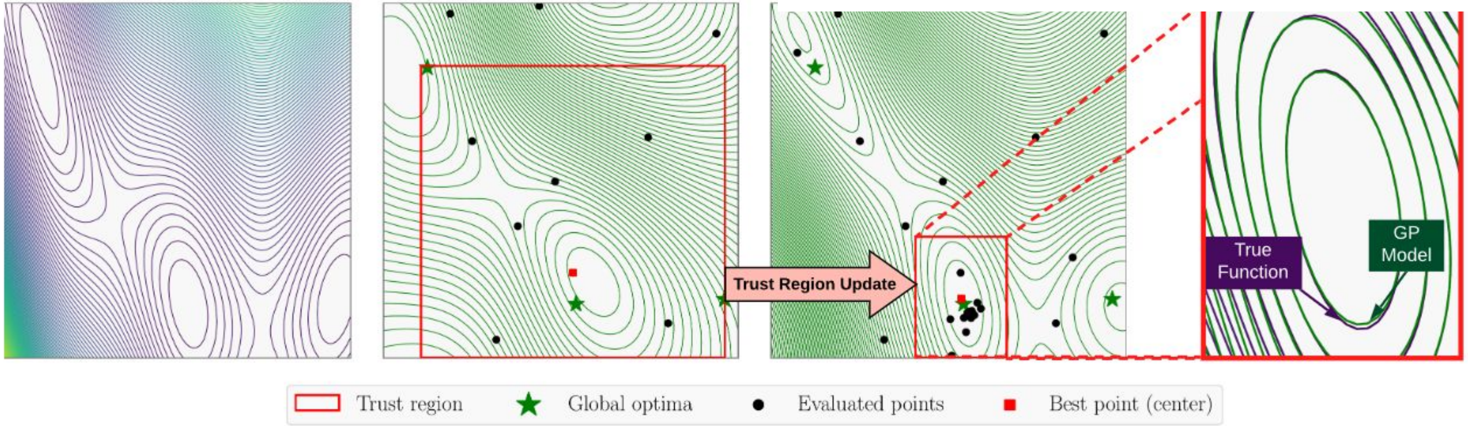
Global Stochastic Search

TuRBO: Bayesian Optimization.

- Find a list of approximate local minima.
- Explore more of the optimization space.
- Later, pick refined minima which are robust.





Local Stochastic Refinement

SAA: Monte Carlo Sampling

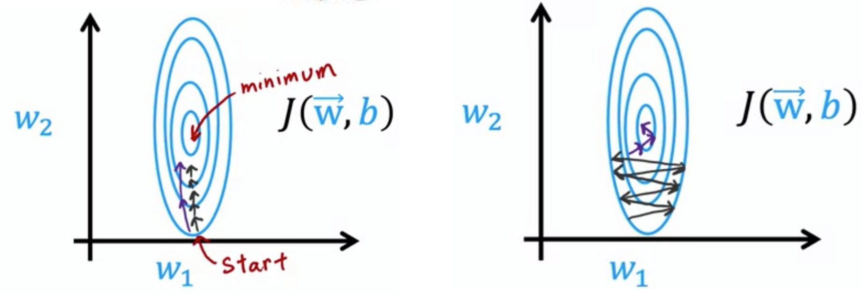
$$f_{\mathrm{SAA}}(\mathbf{x}) = \frac{1}{N_{\mathrm{SAA}}} \sum_{i=1}^{N_{\mathrm{SAA}}} f_{\mathrm{stoc}}(\mathbf{x} + \boldsymbol{u}_i)$$

- Deterministic local search
  (BFGS, other methods)
- Resample and check

Expensive, less evaluations

Adam: Stochastic Gradient

$$\boldsymbol{g}^k = \frac{1}{N_A} \sum_{i=1}^{N_A} \nabla f(\mathbf{x}_k + \boldsymbol{u}_{k,i})$$



$$\mathbf{x}_{k+1} = \mathbf{x}_k - \eta_k \hat{\boldsymbol{m}}_k / (\sqrt{\hat{\boldsymbol{v}}_k} + \epsilon_A),$$

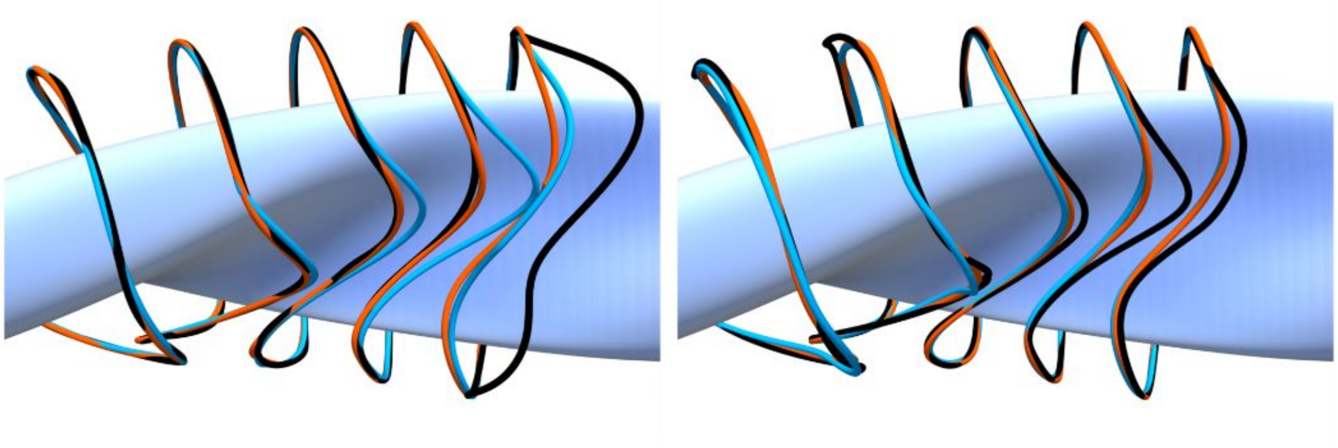
Cheap, many evaluations

Results: W7-X

Glas et al. tried these techniques on Wendelstein 7-X configuration.

Global search: 15 new stochastic local minima.

Local refinement: 8 successful coilsets with different engineering properties.



(a) DTuRBO-AdamCV 5mm. (b) DTuRBO-AdamCV 10mm.

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Future Work

- Apply these techniques to Stage 1 to do global search.
  - Rough objectives
  - Lots of new stellarator shapes
  - Make room for special features: divertors, controlled chaos

- Apply these techniques to coil placement and alignment.

Thank you!

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